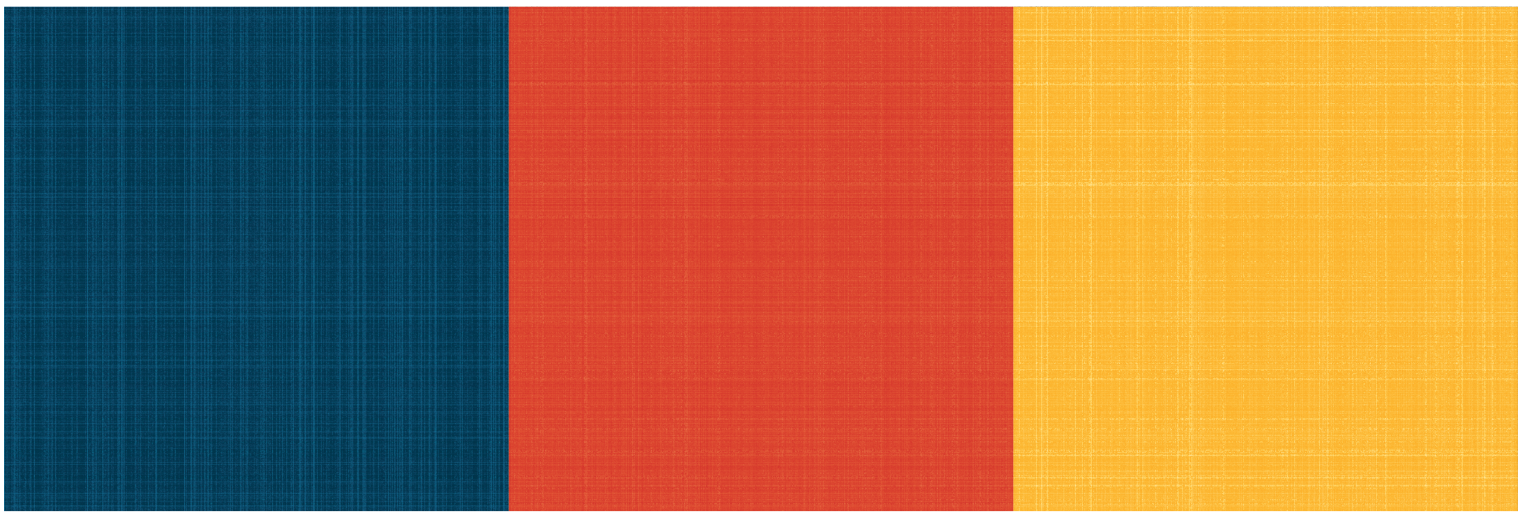
The 3rd Conference on

# Surfaces, Analysis, and Numerics



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Korea University
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# Presentation

*The 3rd Conference on Surfaces, Analysis, and Numerics* is the third installment of a series of conferences held in Korea in the field of differential geometry, covering various topics including, but not limited to, surface theory, geometric analysis, discrete differential geometry, and numerical analysis. The series of conferences aims to bring together both young and established mathematicians from around the world to facilitate the exchange of ideas and the propogation of results. In particular, the series puts an emphasis on providing a platform for international mathematicians to meet and engage Korean mathematicians.

We are thankful to all the speakers and participants for making this conference vibrant and vigorous. We hope that you enjoy the conference to its fullest.

## Organizing committee


Joseph Cho
TU Wien

Junseok Kim
Korea University

Wayne Rossman
Kobe University

Seong-Deog Yang
Korea University

Masashi Yasumoto
Tokushima University

| | 20 (Mon) | 21 (Tue) | 22 (Wed) | 23 (Thu) |
|---|---|---|---|---|
| 10:00 | | **Kyeongsu Choi** (KIAS) | **Seoung Dal Jung** (Jeju Nat'l Univ) | **Wonjoo Lee** (Korea Univ) |
| 10:45 | Registration | *Break* | *Break* | **Yoshiki Jikumaru** (Kyushu Univ) |
| 11:00 | **Xiaoxiang Chai** (KIAS) | **Albert Wood** (Nat'l Taiwan Univ) | **Denis Polly** (Kobe Univ) | *Break* |
| 11:30 | **Eungbeom Yeon** (Pusan Nat'l Univ) | **Yuichiro Sato** (Kogakuin Univ) | **Shintaro Akamine** (Nihon Univ) | **Yibao Li** (Xi'an Jiaotong University) |
| 12:00 | *Lunch Break* | *Lunch Break* | *Lunch Break* | **Nicolau Sarquis Aiex** (Nat'l Taiwan Normal Univ) |
| 14:00 | **Juncheol Pyo** (Pusan Nat'l Univ) | **Hao Chen** (ShanghaiTech Univ) | **Shoichi Fujimori** (Hiroshima Univ) | |
| 14:45 | *Break* | *Break* | *Break* | |
| 15:00 | **Beomjun Choi** (POSTECH) | **Natsuo Miyatake** (Kyushu Univ) | **Pradip Kumar** (Shiv Nadar Univ) | |
| 15:30 | **Joseph Cho** (TU Wien) | **Eriko Shinkawa** (Tohoku Univ) | **Poster Presentation** & *Break* | |
| 16:00 | *Break* | *Break* | | |
| 16:30 | **Miyuki Koiso** (Kyushu Univ) | **Hisashi Naito** (Nagoya Univ) | **Mahan Mj** (TIFR) | |
| 17:15 | | | | |

# Abstracts of Plenary Talks

**Hao Chen**
*ShanghaiTech University*

### The devils in the details when gluing saddle towers

The node opening technique is a powerful tool developed by Traizet for constructing minimal surfaces. It has been very successful in gluing catenoids into minimal surfaces. Actually, it was first applied to glue saddle towers into minimal surfaces, but that construction has met with many restrictions. I will give a detailed account of our recent and ongoing works that glue saddle towers into various types of minimal surfaces. It reveals many devils in the details of the technique, leading to many new examples whose existence was (some still is) subtle to prove.

**Kyeongsu Choi**
*Korea Institute for Advanced Study*

### Noncollapsed translating solitons of mean curvature flow

The mean curvature flow is an evolution of hypersufaces satisfying a geometric heat equation. If the flow is closed then it develops singularities in finite time, and its blow-ups at a singularity converge to ancient flows. Such limit ancient flows are usually self-similar flows or converge to self-similar flows. To be precise, a limit ancient looks like a shrinking soliton at the first glance, but one can also find translating soliton like parts of the ancient flow. In this talk, we discuss about how to find self-similar flows at blow-up limits especially for noncollapsed flows. Also, we talk about classification of noncollapsed shrinking and translating solitons and singularity examples where we can actually obtain the self-similar limits.

**Shoichi Fujimori**
*Hiroshima University*

### Nonorientable maximal surfaces with one end in the Lorentz-Minkowski 3-space

We give a construction for nonorientable maximal surfaces with one end in Lorentz-Minkowski 3-space. We show some existence results for surfaces of this kind with high genus. This is joint work with Shin Kaneda.

**Seoung Dal Jung**
*Jeju National University*

### Frölicher type inequality on foliations

On a compact Riemannian manifold, the de Rham cohomology group is very important cohomological invariant. While in the compact complex manifold, the Dolbeault cohomology is the corresponding invariant. In particular, in the case of Kähler manifold, the Dolbeault cohomology groups give a decomposition of the de Rham cohomology, called Hodge decomposition. But the Hodge decomposition holds no more true, in general, for non-Kähler manifold. Actually, on a non-Kähler manifold, there is an inequality, called the Frölicher inequality.

On the other hand, other important tools to study the geometry of compact complex (especially, non-Kähler) manifolds are the Bott-Chern cohomology and Aeppli cohomology and the Frölicher type inequality. In this talk, we discuss the above properties (basic Hodge decomposition, basic Frölicher type inequality, etc.) on foliations.

**Miyuki Koiso**
*Kyushu University*

## Developable surfaces with curved folds and their continuous isometric deformations to planar regions

A developable surface in the three-dimensional Euclidean space is a surface which is isometric to a planar region, that is, there exists a bijective mapping from a developable surface to a planar region which preserves the length of all curves on the surface. If the considered surface is smooth, then it is a surface with zero Gaussian curvature. In this talk, we study developable surfaces with curved folds, which are naturally appear as origami works and have many applications in manufacturing objects. We discuss the existence of continuous isometric deformations from such a surface to a planar region. As an example, we discuss a variational problem which maximizes the enclosed volume for "pillow boxes" which are isometric to a given flat topological sphere with four "vertexes".

**Mahan Mj**
*Tata Institute of Fundamental Research*

## Percolation on hyperbolic groups

We study first passage percolation (FPP) in a Gromov-hyperbolic group $G$ with boundary equipped with the Patterson-Sullivan measure. We associate an i.i.d. collection of random passage times to each edge of a Cayley graph of $G$, and investigate classical questions about asymptotics of first passage time as well as the geometry of geodesics in the FPP metric. Under suitable conditions on the passage time distribution, we show that the 'velocity' exists in almost every direction, and is almost surely constant by ergodicity of the $G$-action on the boundary. For every point on the boundary, we also show almost sure coalescence of any two geodesic rays directed towards the point. Finally, we show that the variance of the first passage time grows linearly with word distance along word geodesic rays in every fixed boundary direction. No special background will be assumed. This is joint work with Riddhipratim Basu.

**Hisashi Naito**
*Nagoya University*

### Trivalent discrete surfaces and carbon structures

We talk about geometry of trivalent discrete surfaces in 3-dimensional Euclidean space modeled on carbon structures. In particular, curvatures of trivalent discrete surfaces and convergence of subdivisions of them surfaces will be discussed, We will also touch upon the study of the relationship between curvature and physical properties for a model of graphene with defects. If time permits, although they are not directly related to the title. I will mention ongoing studies on physics and material properties using data analysis and machine learning.

**Juncheol Pyo**
*Pusan National University*

### Minimal surfaces and solitons of the mean curvature flow

Minimal surfaces have been extensively studied in differential geometry. Translating solitons and self shrinkers are solitons of the mean curvature flow. They are not only special solutions of the MCF but blow-up models of singularities of MCF. In this talk, we compare some properties of minimal surfaces and solitons of MCF with respect to Bernstein type theorem, half-space type theorem, and free boundary surfaces in a ball. More precisely, we prove that the rigidity results when a graphical translator moves in a non-vertical direction. Secondly, we prove that complete translating solitons can lie on the upper part of a hyperplane and cannot lie on the lower part of it. Finally, we prove that any graphical self-shrinker with free boundary in a ball is a flat disk passing through the center of the ball. These are joint work with Daehwan Kim, Sangwoo Park, Yuan Shyong Ooi, and John Ma.

# Abstracts of Short Communications

**Nicolau Sarquis Aiex**
*National Taiwan Normal University*

### Singularities of minimal hypersurfaces with bounded index

We prove a quantitative estimate on the size of a tubular neighbourhood of the singular set of a minimal hypersurface. This is a generalization of the work of Naber-Valtorta in the case of area minimizing hypersurfaces. These quantitative estimates control not only the size of the singular set but also the behaviour of the hypersurface near singularities. We will discuss motivation and the main idea of the proof.

**Shintaro Akamine**
*Nihon University*

### Extension of Krust theorem and deformations of minimal surfaces

In the minimal surface theory, the Krust theorem asserts that if a minimal surface in the Euclidean 3-space is the graph of a function over a convex domain, then each surface of its associated family is also a graph. The same is true for maximal surfaces in the Minkowski 3-space.

In this talk, we prove a Krust-type theorem for deformation families including various important deformations of minimal and maximal surfaces such as the associated family and the Lopez-Ros deformation and the Calabi-type duality correspondence. We also prove another Krust-type theorem which does not assume the convexity assumption. The results are proved based on the recent progress of planar harmonic mapping theory.

This talk is mainly based on the joint work with Hiroki Fujino (Nagoya University, Japan).

**Xiaoxiang Chai**
*Korea Institute for Advanced Study*

### Willmore type inequalities in geodesic balls of hyperbolic space

Starting from a convex surface in hyperbolic space supported on the geodesic sphere, the inverse mean curvature flow converges to a totally geodesic disk. Using this convergence result, we show that a Willmore type quantity is monotonically decreasing and we show that the quantity is bounded below.

**Joseph Cho**
*TU Wien*

### Discrete isothermic tori via Darboux transformations

We discuss the conditions to obtain closed Darboux transforms of discrete isothermic surfaces. We test the robustness of our theory by creating concrete examples. This talk is based on the joint work with Katrin Leschke and Yuta Ogata.

**Beomjun Choi**
*Pohang University of Science and Technology*

### Liouville theorem for surfaces translating by powers of Gauss curvature

We discuss the classification of entire solutions to degenerate Monge-Ampere equations $\det D^2 u = (1 + |Du|^2)^\beta$ on $\mathbb{R}^2$ for $\beta < 0$. The graphs of such solutions correspond to the translating solitons to the sub-affine-critical powers of Gauss curvature.

In the critical case $\det D^2 u = 1$, the celebrated result by Jorgens, Calabi and Pogorelov shows every entire solution must be a convex paraboloid and hence the level sets are homothetic ellipses. In our cases, level sets are modeled by convex closed curves which self-similarly shrink under the flows by sub-affine-critical powers of curvature. Then we study the moduli space of solutions for each prescribed asymptotics. This is a joint work with K. Choi and S. Kim.

**Yoshiki Jikumaru**
*Kyushu University*

### On the Michell truss-like structures based on discrete isothermic surfaces

It is known that a discrete isothermic surface can be regarded as a discrete shell membrane in equilibrium with purely tangential internal forces. In this talk, we show that this characteristic can be exploited to generate various type of truss structures in architectural design which possess some "nice" mechanical properties. In particular, we discuss a truss structure analogous to the classical Michell truss and show the relation with discrete log-aesthetic curves.

**Pradip Kumar**
*Shiv Nadar University*

### Swallowtails and cone-like singularities on a maxface

Maximal surfaces in 3-dimensional Lorentz space $\mathbb{E}^3_1$ of signature $(-++)$ are spacelike immersions that maximize area locally. They are similar to the minimal surfaces in $\mathbb{R}^3$, as both are zero mean curvature surfaces, and both can be constructed using many similar methods. But in contrast to the minimal surface, in the generalized maximal surfaces, non-isolated singularities appear. In this talk- we recall various types of singularities that appear on the generalized maximal immersions. We will discuss the criterion to determine the nature of singularities from the singular Björling data. Moreover, we will discuss the relationship between cone-like singularities and swallowtails. The last, we will explore the problem of when we can interpolate two curves (with the prescribed nature of singularity) by generalized maximal immersions.

**Wonjoo Lee**
*Korea University*

### Rotationally invariant zero mean curvature surfaces in the three dimensional light cone space

In the three dimensional Euclidean space, catenoids are nontrivial and are the only minimal surfaces that are rotationally invariant. However, unlike the three dimensional Euclidean space, there are three types of rotations in the three dimensional light cone space. In this talk, we find zero mean curvature surfaces that are invariant to three rotations in the three dimensional light cone space, and visualize them by using stereographic projection. This talk is based on the joint work with Joseph Cho, So Young Kim, Dami Lee, and Seong-Deog Yang.

**Yibao Li**
*Xi'an Jiaotong University*

### Multiscale topology optimization method for lattice materials

In this talk, we will introduce an efficient multiscale topology optimization method for lattice materials. In macro-scale, we present a second-order unconditionally energy stable schemes for the topology optimization problem. Using porous media approach, our objective functional composes of five terms including mechanical property, Ginzburg-Landau energy, two penalized terms for solid and the volume constraint. A Crank-Nicolson method is proposed to discrete the coupling system. We prove that our proposed scheme is unconditionally energy stable. In macro-scale, we propose a simple volume merging method for triply periodic minimal structure. A modified Allen–Cahn type equation with a correction term is proposed. The mean curvature on the surface will be constant everywhere at the equilibrium state. Computational experiments are presented to demonstrate the efficiency of the proposed method.

**Natsuo Miyatake**
*Kyushu University*

### Generalizations of Hermitian-Einstein equation of cyclic Higgs bundles

We introduce some generalizations of the Hermitian-Einstein equation for diagonal harmonic metrics on cyclic Higgs bundles, including a generalization using subharmonic functions. When the coefficients are all smooth, we prove the existence, uniqueness, and convergence of the solution of their heat equations with Dirichlet boundary conditions. We also generalize two basic inequality estimates for solutions of the Hermitian-Einstein equation of cyclic Higgs bundles.

**Denis Polly**
*Kobe University*

### Rotational cmc surfaces in space forms

We describe rotational constant mean curvature surfaces in space forms through the lens of sphere geometry. This yields a classification of all cmc surfaces in spherical and hyperbolic space in terms of explicit parametrizations in terms of elliptic functions. We then describe how our method extends to the class of elliptic linear Weingarten surfaces.

**Yuichiro Sato**
*Kogakuin University*

### $d$-minimal surfaces in isotropic 3-space and applications

In affine 3-space endowed with a degenerate metric, we define $\nabla$-minimal surfaces as analogue objects to classical minimal surfaces, where $\nabla$ is a torsion-free, metric connection with respect to the degenerate metric. In particular, when $\nabla$ is the canonical connection $d$ of affine 3-space, $d$-minimal surfaces coincide with isotropic minimal surfaces in isotropic 3-space. In this talk, we give a representation formula of Weierstrass type for $d$-minimal surfaces, and claim that $d$-minimal surfaces allow to have isolated singular points. Moreover, we give two applications. Firstly, we show that $d$-minimal surfaces and spacelike flat zero mean curvature surfaces in Minkowski 4-space are in one-to-one correspondence. Secondly, we give a visualization of a deformation of zero mean curvature surfaces in Minkowski 4-space.

**Eriko Shinkawa**
*Tohoku University*

### Geometry of anisotropic double crystals and classification of some examples

Double bubbles are a mathematical model of soap bubbles. The energy functional is the total area of the surface. On the other hand, when we think about a mathematical model of anisotropic substances like crystals, we need to consider the energy density function depending on the normal direction of the surface. The energy density function is called an anisotropic energy density function, and its sum (integral) over the surface is called anisotropic energy.

In this study, we extend the double bubble problem to an anisotropic problem, that is, we minimize the anisotropic energy instead of the surface area. We derive the first variation formulas of the energy functional. For $n = 1$ and a kind of special energy density function, we classify the double crystals in terms of symmetry and the given areas.

**Albert Wood**
*National Taiwan University*

### Cohomogeneity-one Lagrangian mean curvature flow

Lagrangian mean curvature flow is the name given to the observation that the class of Lagrangian submanifolds of Calabi-Yau manifolds is preserved under mean curvature flow. This observation gave rise to a conjecture of Thomas-Yau, which states that assuming a stability condition on the Lagrangian, the flow should converge to a unique volume-minimising representative. Since mean curvature flow typically forms finite-time singularities, a surgery procedure must be defined to resolve the conjecture, and an understanding of the possible singularity models is a vital first step.

In this work, we study Lagrangians that are invariant with respect to a group action respecting the Calabi-Yau structure with $(n-1)$-dimensional orbits. Such Lagrangians must be contained in an $(n+1)$-dimensional submanifold, a level set of the moment map, and taking the symplectic quotient produces a curve in a 2-manifold. Lagrangian mean curvature flow may therefore be studied via a related curve shortening flow, which we show does not depend on the group action. By this method, we are able to classify cohomogeneity-one shrinking and expanding solitons, as well as fully classify singularities in the case of the zero level set.

**Eungbeom Yeon**
*Pusan National University*

## Minimal hypersurfaces in Schwarzschild Riemannian $n$-manifolds

Affected by the spherical mass body, there are very few complete minimal hypersurfaces in Schwarzschild Riemannian manifolds. We review some of the non-existence results in Schwarzschild Riemannian manifolds and introduce Frankel's property for free boundary minimal hypersurfaces in the Riemannian Schwarzschild $n$-manifolds. Furthermore, we extend this property to some class of asymptotically flat manifolds where its scalar curvature is no longer positive. This talk includes the contents form a joint work with Dr. Jaehoon Lee.